Reciprocal Identities and the Calculator

The calculator can only calculate $\sin\theta$, $\cos\theta$, and $\tan\theta$.
To calculate $\csc\theta$, $\sec\theta$, and $\cot\theta$ we use the reciprocal identities.

$\sin\theta = \frac{1}{\csc\theta}$ $\quad$ $\cos\theta = \frac{1}{\sec\theta}$ $\quad$ $\tan\theta = \frac{1}{\cot\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\quad$ $\sec\theta = \frac{1}{\cos\theta}$ $\quad$ $\cot\theta = \frac{1}{\tan\theta}$

Note that if, for instance, $\tan\theta = 0$, then $\cot\theta = DNE$ since division by zero is illegal.

Notes

Examples of Reciprocal Identities

Example (Find $\cos\theta$ if $\sec\theta = \frac{5}{3}$)

Since $\cos\theta$ is the reciprocal of $\sec\theta$,

$$\cos\theta = \frac{3}{5}$$

Example (Find $\sin\theta$ if $\csc\theta = -\frac{\sqrt{12}}{6}$)

$$\begin{aligned}\sin\theta &= -\frac{6}{\sqrt{12}} \\ &= -\frac{2\cdot 3}{\sqrt{4\cdot 3}} \\ &= -\frac{3}{\sqrt{3}}\end{aligned}$$

Since $\sin\theta$ is the reciprocal of $\csc\theta$, we have

Note: we could have simplified the value of $\csc\theta$ first.

Notes

Quadrants and Signs

- Using the definitions of the trig functions, r is the distance from $(0,0)$ to (x,y). Distance is never negative, so $r > 0$.
- x and y can be positive or negative, depending on the quadrant.
- Therefore the sign of the trig function values will depend on the quadrant the angle is in.

Quadrant of θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
(x+, y+) I	+	+	+	+	+	+
(x-, y+) II	+	-	-	-	-	+
(x-, y-) III	-	-	+	+	-	-
(x+, y-) IV	-	+	-	-	+	-

Notes

Using Quadrants and Signs

Example (What is the quadrant(s) of θ if $\sin\theta > 0$ and $\tan\theta < 0$)

- $\sin\theta$ is positive only in quadrants I and II
- $\tan\theta$ is negative only in quadrants II and IV
- Answer: θ must lie in quadrant II

Notes

Finding All Function Values given One Value and the Quadrant

Example (θ lies in quadrant III and $\sin\theta = -\frac{2}{3}$)

1. $y = -2$, since we're in Q III and $r = 3$, but we need x:
   $x^2 + 2^2 = 3^2$
   or $x^2 = 9 - 4 = 5$
   or simply $x = \pm\sqrt{5}$
   But which; positive or negative?
2. θ lies in Q III, therefore x is negative, and
   $x = -\sqrt{5}$
3. $\sin\theta = -\frac{2}{3}$
   $\cos\theta = -\frac{\sqrt{5}}{3}$
   $\tan\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$
   $\csc\theta = -\frac{3}{2}$
   $\sec\theta = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$
   $\cot\theta = \frac{\sqrt{5}}{2}$

Notes

Homework

Calculate the six trigonometric function values for the following cases:

1. θ is in quadrant IV, and $\sin\theta = -\frac{1}{3}$
2. θ is in quadrant II, and $\tan\theta = -2$
3. θ is in quadrant IV, and $\csc\theta = -\frac{\sqrt{5}}{2}$
4. θ is in quadrant III, and $\sec\theta = -\frac{5}{4}$
5. θ is in quadrant I, and $\cot\theta = \frac{2}{9}$
6. θ is in quadrant III, and $\cos\theta = -\frac{5}{7}$

Notes