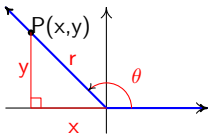
Distance Between Two Points

To calculate the distance between two points $A = (x_1, y_1)$ ad $B = (x_2, y_2)$, we use the Euclidean formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Consider an angle drawn in standard position. Pick a point P on the terminal vertex.



Then the distance r from $P(x, y)$ to the origin $(0, 0)$ is

$$\begin{aligned} r &= \sqrt{(x-0)^2 + (y-0)^2} \\ &= \sqrt{x^2 + y^2} \end{aligned}$$

Notes

The Trigonometric Functions

We can define the 6 trigonometric functions of angle θ in terms of x, y, and r.

Definition (Trig functions)

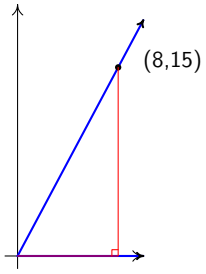
$\sin\theta = \frac{y}{r}$ $\quad \cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}, \quad (x \neq 0)$

$\csc\theta = \frac{r}{y}, \quad (y \neq 0)$ $\quad \sec\theta = \frac{r}{x}, \quad (x \neq 0)$ $\quad \cot\theta = \frac{x}{y}, \quad (y \neq 0)$

Notes

Finding Function Values of an angle

The terminal side of angle θ in standard position passes through the point $(8, 15)$. Find the value of the six trigonometric functions of θ.



Step 1:

$$\begin{aligned} r &= \sqrt{8^2 + 15^2} \\ &= \sqrt{64 + 225} \\ &= \sqrt{289} \\ &= 17 \end{aligned}$$

Step 2:
Use x, y, and r:

$\sin\theta = \frac{15}{17}$

$\cos\theta = \frac{8}{17}$

$\tan\theta = \frac{15}{8}$

$\csc\theta = \frac{17}{15}$

$\sec\theta = \frac{17}{8}$

$\cot\theta = \frac{8}{15}$

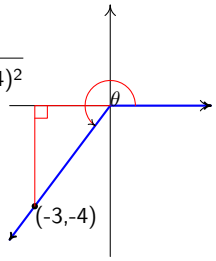
Notes

Finding Function Values of an angle

The terminal side of angle θ in standard position passes through the point $(-3, -4)$. Find the value of the six trigonometric functions of θ.

Step 1:

$$\begin{aligned} r &= \sqrt{(-3)^2 + (-4)^2} \\ &= \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$



Step 2:
Use x, y, and r:

$\sin\theta = \frac{-4}{5}$

$\cos\theta = \frac{-3}{5}$

$\tan\theta = \frac{-4}{-3} = \frac{4}{3}$

$\csc\theta = \frac{5}{-4}$

$\sec\theta = \frac{5}{-3}$

$\cot\theta = \frac{-3}{-4} = \frac{3}{4}$

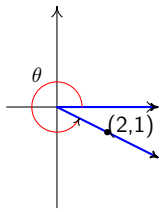
Notes

Finding Function Values of an angle

Find the value of the six trigonometric functions of θ, if the terminal side is defined by $x + 2y = 0$, $x \geq 0$

Step 1:
Find a point on the terminal line.
We can use any value for x except 0;
Let's use $x = 2$

$$\begin{aligned} x + 2y &= 0, x \geq 0 \\ 2 + 2y &= 0 \\ 2y &= -2 \\ y &= -1 \end{aligned}$$



Step 2:

$$\begin{aligned} r &= \sqrt{2^2 + (-1)^2} \\ &= \sqrt{5} \end{aligned}$$

Step 3:
Use x, y, and r:

$\sin\theta = \frac{-1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

$\cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\tan\theta = \frac{-1}{2}$

$\csc\theta = \frac{\sqrt{5}}{-1} = -\sqrt{5}$

$\sec\theta = \frac{\sqrt{5}}{2}$

$\cot\theta = \frac{2}{-1} = -2$

Notes

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