The Sandwich Theorem

In some cases, it is not easy (or possible) to find the limit of a function directly.
In those cases, we can use the Sandwich Theorem to deduce what the limit is.
We *sandwich* our function between two functions whose limits are known.
If those two functions both have the same limit, our function must be trapped in between.

Notes

The Sandwich Theorem (Formal Statement)

Theorem
If $g(x) \leq f(x) \leq h(x)$ *for all* $x \neq c$ *in some interval about* c*, and*

$$\lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$$

then

$$\lim_{x \to c} f(x) = L$$

Notes

Example of Sandwich Theorem

Example ($\lim_{x \to 0} x^2 \sin(1/x)$)

- The range of $\sin(1/x)$ is $[-1, 1]$
- $|x^2 \sin \frac{1}{x}| = |x^2| \cdot |\sin \frac{1}{x}| \leq |x^2| \cdot 1 = x^2$
- Therefore
$$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$$
- $\lim_{x \to 0} -x^2 = 0$, and $\lim_{x \to 0} x^2 = 0$
- Therefore $\lim_{x \to 0} x^2 \sin(1/x) = 0$

Notes

Polynomial Long Division

Example $((6x^3 - 2x^2 + 5x + 9) \div (3x + 2))$

Why use long division? Synthetic division only works when dividing by $(x \pm c)$

Answer is $2x^2 - 2x + 3$ with a remainder of 3

Example $((4x^4 - 3x^3 + 7x^2 - x + 10) \div (x^2 - 1))$

Answer is $4x^2 - 3x + 11$ with a remainder of $-4x + 21$

Notes

Homework

Section 2.2 QR (page 75): (6 problems)

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6

7 a

b

8 a

b

9 a

b

10 a

b

Notes

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