

Left and Right handed Limits

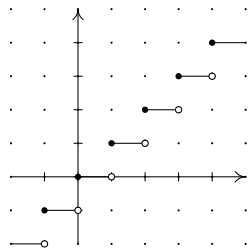


Figure: $y = \text{int } x$

- Right handed:
 $\lim_{x \rightarrow c^+} f(x)$ Limit of f as x approaches from the right.
 $\lim_{x \rightarrow 2^+} \text{int } x = 2$
- Left handed: $\lim_{x \rightarrow c^-} f(x)$
 Limit of f as x approaches from the left.
 $\lim_{x \rightarrow 2^-} \text{int } x = 1$

Notes

General Limits

Theorem

A function $f(x)$ has a limit as x approaches c if and only if the left- and right-hand limits exist and are equal.

$$\lim_{x \rightarrow c} f(x) = L \Leftrightarrow \lim_{x \rightarrow c^+} f(x) = L = \lim_{x \rightarrow c^-} f(x)$$

Notes

Examples of left and right hand limits

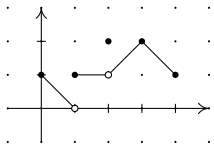


Figure: The (piecewise) function $f(x)$

- $\lim_{x \rightarrow 0^+} f(x) = 1$
- $\lim_{x \rightarrow 1^-} f(x) = 0$
- $\lim_{x \rightarrow 1^+} f(x) = 1$
- $\lim_{x \rightarrow 1} f(x) = \text{DNE}$
- $\lim_{x \rightarrow 2^-} f(x) = 1$
- $\lim_{x \rightarrow 2^+} f(x) = 1$
- $\lim_{x \rightarrow 2} f(x) = 1$ even though $f(2) = 2$
- $\lim_{x \rightarrow 3^-} f(x) = 2$
- $\lim_{x \rightarrow 3^+} f(x) = 2$
- $\lim_{x \rightarrow 3} f(x) = 2 = f(3)$
- $\lim_{x \rightarrow 4^-} f(x) = 1$

Notes

Homework

Section 2.1 (page 66): (9 problems)

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30 (Need to sketch graph. Discuss left and right hand limits.)

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Notes

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