Mean Value Theorem

Theorem (Mean Value Theorem)

If $y = f(x)$ is continuous over the interval $[a, b]$ and differentiable at every point of its interior (a, b), then there exists a point c in (a, b) such that:

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Notes

Exploring the Mean Value Theorem

Show that the function $f(x) = x^2$ satisfies the Mean Value Theorem on the interval $[0, 2]$.

Then find the point c.

- $f(x)$ is differentiable and continuous over the interval $[0, 2]$
- Solve

$$f'(c) = \frac{f(2) - f(0)}{2 - 0}$$

if $f'(c) = 2x$

Notes

Exploring the Mean Value Theorem (cont)

For each function, explain why the function fails to satisfy the conditions over the interval $[-1, 1]$.

- $y = \sqrt{x^2} + 1$

Cusp at x = 0, so not differentiable at x=0

- $y = \begin{cases} x^3 + 3 & x < 1 \\ x^2 + 1 & x \geq 1 \end{cases}$

Discontinuity at x = 1

Notes

Physical Interpretation

Consider $\frac{f(b)-f(a)}{b-a}$:

This is the average rate of change over the interval $[a, b]$

$f'(c)$ is the instantaneous rate of change.

The mean Value Theorem says that at some point, the average and instantaneous rates of change must be equal.

Notes

Physical Example

Example

If a car accelerating from zero takes 8 sec to go 352 ft, its average velocity for the 8-sec interval is 352/8=44 ft/sec. At some point during the acceleration, the theorem says, the speedometer must read exactly 44ft/sec.

If we know that the car reaches a top speed of 50ft/sec, then how at least many times must the speedometer have read exactly 44ft/sec?

How fast is ft/sec measured as miles per hour?

Notes

Homework

Sect 4.2: 1 - 13 Odd

1 a Yes/No
  b $c =$
3 a Yes/No
  b $c =$
5 a Yes/No
  b $c =$
7 a Yes/No
  b $c =$
9 a $y =$
  b $y =$
11
13

Notes