Homework

Section 2.1 Quick Review (page 65): (14 problems)

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Section 2.1 (page 66):

1.
2.
3.
4.

Notes

Overview

- Notation
- Graphical Limits
- Properties of limits

Notes

Notation

Definition

Assume that the function f is defined in the neighbourhood of c, and let c and L be real numbers.
The function f has limit L as x approaches c if the distance between $f(x)$ and L becomes arbitrarily small when x comes close to c.

Notation

$$\lim_{x \to c} f(x) = L$$

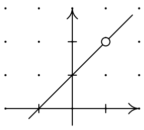
Pronounced as "the limit of f of x as x goes to c equals L".

Example

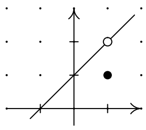
$$\lim_{h \to 0} (64 + 16h) = 64$$

Notes

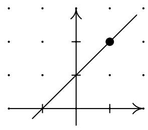
Graphical Examples



$f(x) = \frac{x^2-1}{x-1}$



$f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$



$f(x) = x + 1$

Notes

Properties of Limits

Given only two facts, we can calculate the limits of all polynomial and rational functions:

1. $\lim_{x \to c}(k) = k$: the limit of a constant is that constant.
2. $\lim_{x \to c}(x) = c$: the limit of the identity function.

Notation

Assume that L, M, c, and k are real numbers and f and g are two functions so that:

$\lim_{x \to c} f(x) = L$

$\lim_{x \to c} g(x) = M$

. . . Continued next slide

Notes

Properties of Limits (cont)

- $\lim_{x \to c}(f(x) + g(x)) = L + M$ (Sum Rule)
- $\lim_{x \to c}(f(x) - g(x)) = L - M$ (Difference Rule)
- $\lim_{x \to c}(f(x) \cdot g(x)) = L \cdot M$ (Product Rule)
- $\lim_{x \to c}(k \cdot f(x)) = k \cdot L$ (Constant Multiple Rule)
- $\lim_{x \to c}(\frac{f(x)}{g(x)}) = \frac{L}{M}, M \neq 0$ (Quotient Rule)
- $\lim_{x \to c}(f(x))^{r/s} = L^{r/s}$, assuming $L^{r/s}$ is a real number (Power Rule)

Notes

Using the Properties

Example $(x^3 + 4x^2 - 3)$

$$\begin{aligned} \lim_{x\to c}(x^3 + 4x^2 - 3) &= \lim_{x\to c}(x^3) + \lim_{x\to c}(4x^2) - \lim_{x\to c} 3 \\ &= c^3 + 4c^2 - 3 \end{aligned}$$

Notes

Using the Properties

Example $(\frac{x^4+x^2-1}{x^2+5})$

$$\begin{aligned} \lim_{x\to 2}(\frac{x^4 + x^2 - 1}{x^2 + 5}) &= \frac{\lim_{x\to 2}(x^4) + \lim_{x\to 2}(x^2) - \lim_{x\to 2} 1}{\lim_{x\to 2}(x^2) + \lim_{x\to 2}(5)} \\ &= \frac{2^4 + 2^2 - 1}{2^2 + 5} \\ &= \frac{16 + 4 - 1}{4 + 5} \\ &= \frac{19}{9} \end{aligned}$$

Notes

Homework

Section 2.1 (page 66): (4 problems)

5
6
7
8

Notes