

Overview

- The concept of the limit is one of the ideas that distinguishes calculus from algebra and trigonometry/geometry.
- We can use limits to test if a function is continuous.
- We can find limits using a variety of tools:
 - substitution (plug it in)
 - graphical investigation (graph it)
 - algebra (manipulate it)
 - some or all of the above (play with it)

Notes

Average and Instantaneous Speed

Definition (Average Speed)

A body's *average speed* during an interval of time is found by dividing the distance the object covers by the elapsed time.

The unit of measure of average speed is length per unit time. I.e. km/hr, or mph, or ft/s, or similar appropriate units.

Example (Average Speed)

A rock breaks loose from the top of a tall cliff. What is its average speed during the first 2 seconds of fall?

- An object dropped from rest on the earth will fall $f(t) = 16t^2$ ft in the first t seconds.
- The average speed $s = \frac{\Delta y}{\Delta t} = \frac{16(2)^2 - 16(0)^2}{2 - 0} = \frac{64}{2} = 32$ ft/s

Notes

Instantaneous Speed

So how fast is this rock going at exactly 2 seconds?

- Calculate the average speed between $t = 2$ and the slightly later time $t = 2 + h$
- $s = \frac{\Delta y}{\Delta t} = \frac{16(2+h)^2 - 16(2)^2}{(2+h) - (2)} = \frac{16(2+h)^2 - 16(2)^2}{h}$
- Why won't it work just to set $h = 0$?
- How can we get a good idea of what is happening when h is close to 0?

Notes

Instantaneous Speed (cont) - Numerically

$$s = \frac{\Delta y}{\Delta t} = \frac{16(2+h)^2 - 16(2)^2}{h}$$

| Time interval h | Average speed $\Delta y/\Delta t$ (ft/sec) |
|-------------------|---|
| 1 | 80 |
| 0.1 | 65.6 |
| 0.01 | 64.16 |
| 0.001 | 64.016 |
| 0.0001 | 64.0016 |
| 0.00001 | 64.00016 |

Notes

Instantaneous Speed (still cont) - Algebraically

What happens if we try to work the limit out algebraically?

$$\begin{aligned}\frac{\Delta y}{\Delta t} &= \frac{16(2+h)^2 - 16(2)^2}{h} \\ &= \frac{16(4 + 4h + h^2) - 64}{h} \\ &= \frac{64 + 64h + 16h^2 - 64}{h} \\ &= \frac{64h + 16h^2}{h} \\ &= 64 + 16h\end{aligned}$$

Notes

Algebra vs Calculus

- Algebra will let us calculate the average speed.
- Calculus will let us calculate the instantaneous speed.
- Calculus will let us calculate rates of change; how fast are things changing.
- Calculus will let us calculate how far we have traveled, given our speed at any time.
- To do all this we use limits.
- **Today we start Calculus**

Notes

Homework

Section 2.1 Quick Review (page 65): (14 problems)

| | |
|---|----|
| 1 | 6 |
| 2 | 7 |
| 3 | 8 |
| 4 | 9 |
| 5 | 10 |

Section 2.1 (page 66):

| | |
|---|---|
| 1 | 3 |
| 2 | 4 |

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