Graphing Functions

- Ancient history: derivatives used to graph functions
- Modern history:
  - use graphing calculators
  - use derivatives to check graphing program
- Throughout history: use derivatives to solve optimization problems.

Notes

Basic Definitions: Absolute Extrema

Definition (Absolute (Global) Extreme Values)

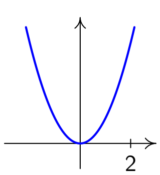
Let f be a function with domain D. Then $f(c)$ is the

- absolute maximum value on D if and only if $f(x) \leq f(c)$ for all x in D.
- absolute minimum value on D if and only if $f(x) \geq f(c)$ for all x in D.

Rough Translation: the absolute maximum is the highest point on the graph, and the absolute minimum is the lowest point on the graph.

Notes

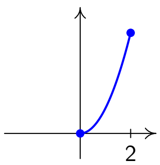
Exploring Absolute Extrema



- Function f: $y = x^2$
- Domain D: $(-\infty, \infty)$
- Absolute extrema on D:
  - Maximum: None
  - Minimum: 0 at x=0

Notes

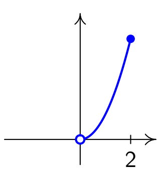
Exploring Absolute Extrema



- Function f: $y = x^2$
- Domain D: $[0, 2]$
- Absolute extrema on D:
  - Maximum: 4 at x=2
  - Minimum: 0 at x=0

Notes

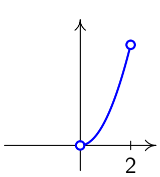
Exploring Absolute Extrema



- Function f: $y = x^2$
- Domain D: $(0, 2]$
- Absolute extrema on D:
  - Maximum: 4 at x=2
  - Minimum: None

Notes

Exploring Absolute Extrema



- Function f: $y = x^2$
- Domain D: $(0, 2)$
- Absolute extrema on D:
  - Maximum: None
  - Minimum: None

Notes

Extreme Value Theorem

Theorem (Extreme Value Theorem)

If f is continuous on a closed interval $[a, b]$, then f has both a minimum and maximum value.

Places to check for extrema:

- Both endpoints
- Interior

Notes

Basic Definitions: Local Extrema

Definition (Local Extreme Values)

Let f be a function, and c be an interior point of the domain. Then $f(c)$ is the

- local maximum value on D if and only if $f(x) \leq f(c)$ for all x in some open interval containing c.
- local minimum value on D if and only if $f(x) \geq f(c)$ for all x in some open interval containing c.

Rough Translation: the local maximum is the highest point in its neighborhood, and the local minimum is the lowest point in its neighborhood.

Note: Endpoints can be local extrema.

Notes

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