Calculating the derivative of $y = \ln x$

- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$

- $\begin{aligned} y &= \ln(x^2 + 3x) \\ y' &= \frac{1}{x^2+3x}(2x+3) \\ &= \frac{2x+3}{x^2+3x} \end{aligned}$

$$\begin{aligned} y &= \ln(\cos x + \sec x) \\ y' &= \frac{1}{\cos x + \sec x}(-\sin x + \tan x \sec x) \\ &= \frac{-\sin x + \tan x \sec x}{\cos x + \sec x} \end{aligned}$$

Notes

Differentiating $y = \log_a x$

To calculate the derivative of $\log_a x$ we can use the change of base formula.

$$\begin{aligned} \log_a x &= \frac{\ln x}{\ln a} \\ &= \frac{1}{\ln a} \cdot \ln x \\ \frac{d}{dx}(\log_a x) &= \frac{1}{\ln a} \cdot \frac{1}{x} \\ &= \frac{1}{x \ln a} \\ \frac{d}{dx}(\log_a u) &= \frac{1}{u \ln a} \cdot \frac{du}{dx} \end{aligned}$$

Notes

Examples

1. $y = \ln(x^2 + x^4)$

   $y' = \frac{2x+4x^3}{x^2+x^4} = \frac{2+4x^2}{x+x^3}$

2. $y = \ln(\frac{1}{x^2})$

   $y' = \frac{-2x^{-3}}{x^{-2}} = \frac{-2}{x}$

3. $y = \log_5(3x^2)$

   $y' = \frac{6x}{3x^2 \ln 5} = \frac{2}{x \ln 5}$

4. $y = \ln(x^2 + \sqrt{x})$

   $y' = \frac{2x + \frac{1}{2\sqrt{x}}}{(x^2+\sqrt{x})}$

Notes