Overview

- Inverse Properties of Logarithms
- Properties of Logarithms and Exponents
- Change of Base formula

Notes

Inverse Properties of Logarithms

Definition

1. Base a: $a^{\log_a x} = x, \log_a a^x = x$ for $a > 1, x > 0$
2. Base e: $e^{\ln x} = x, \ln e^x = x$ for $x > 0$

Examples (Solve for x)

$$\begin{aligned} \ln x &= 3t + 5 \\ e^{\ln x} &= e^{3t+5} \\ x &= e^{3t+5} \end{aligned}$$

$$\begin{aligned} e^{2x} &= 10 \\ \ln e^{2x} &= \ln 10 \\ 2x &= \ln 10 \\ x &= \frac{1}{2} \ln 10 \approx 1.15 \end{aligned}$$

Notes

Properties of Logarithms and Exponents

Definition

For any real numbers $x > 0$ and $y > 0$

	Logarithm	Exponent
Product Rule:	$\log_a xy = \log_a x + \log_a y$	$a^x a^y = a^{x+y}$
Quotient Rule:	$\log_a \frac{x}{y} = \log_a x - \log_a y$	$\frac{a^x}{a^y} = a^{x-y}$
Power Rule:	$\log_a x^y = y \log_a x$	$(a^x)^y = a^{xy}$

Notes

Examples

Example (Solve for t)

$$\begin{aligned} (1.005)^t &= 3 \\ \ln(1.005)^t &= \ln 3 \\ t \ln(1.005) &= \ln 3 \\ t &= \frac{\ln 3}{\ln 1.005} \\ &\approx 220.271 \end{aligned}$$

$$\begin{aligned} e^{0.072t} &= 4 \\ \ln e^{0.072t} &= \ln 4 \\ 0.072t \ln e &= \ln 4 \\ 0.072t &= \ln 4 \\ t &= \frac{\ln 4}{0.072} \end{aligned}$$

Notes

Examples

Example (Solve for y)

$$\begin{aligned} \ln y &= 12x + 7 \\ e^{\ln y} &= e^{12x+7} \\ y &= e^{12x+7} \end{aligned}$$

$$\begin{aligned} \ln(y+7) - \ln 3 &= 3x \\ \ln y + 7 &= 3x + \ln 3 \\ e^{\ln y + 7} &= e^{3x + \ln 3} \\ y + 7 &= e^{3x}(3) \end{aligned}$$

Notes

Change of Base formula

Definition (Changing Bases in a Logarithm)

$\log_a x = \frac{\ln x}{\ln a}$

Examples

- $\log_4 7 = \frac{\ln 7}{\ln 3}$
- $\log_3 x = \frac{\ln x}{\ln 3}$
- $\log_1 216x = \frac{\ln 6x}{\ln 121}$

Notes

Examples

Example (Solve for x)

$$
\begin{aligned}
4^x + 4^{-x} &= 7 \\
4^x - 7 + 4^{-x} &= 0 \\
4^x(4^x - 7 + 4^{-x}) &= 4^x(0) \\
(4^x)^2 - 7(4^x) + 1 &= 0 \\
4^x &= \frac{7 \pm \sqrt{(7^2) - 4(1)(1)}}{2(1)} \quad \text{QuadraticFormula} \\
&= \frac{7 \pm \sqrt{45}}{2} \\
x &= \log_4 \frac{7 \pm \sqrt{45}}{2}
\end{aligned}
$$

Notes

Homework

Section 1.5 (p 45): 33 - 38 (6 problems)

33

34

35

36

37

38

Notes

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