Bellringer - January 22

Remember that

$$\frac{d}{dx}(e^x) = e^x$$

Calculate $\frac{dy}{dx}$ for the following:

1. $y = 4e^x$
2. $y = x^2e^x$
3. $y^2 = 3e^x$

Notes

Finding the Derivative of e^u

Example (Differentiate $y = e^{x+x^2}$)

- Consider the chain rule, where $u = x + x^2$.
- Then $y = e^u$, and $\frac{dy}{dx} = e^u \frac{du}{dx}$
- Therefore $\frac{dy}{dx} = e^{x+x^2}(1 + 2x)$

Example (Differentiate $y = e^{x^3+3x^2}$)

$$\frac{dy}{dx} = e^{x^3+3x^2}\frac{d}{dx}(x^3 + 3x^2)$$

$$\frac{dy}{dx} = e^{x^3+3x^2}(3x^2 + 6x)$$

Notes

What if $y = a^x$, $a \neq e$?

If $a > 0$ and $a \neq 1$ we can use the properties of logarithms to write a^x in terms of e^x and then calculate the derivative.

$$\begin{aligned} a^x &= e^{\ln(a^x)} \\ &= e^{x\ln(a)} \\ \frac{d}{dx}(a^x) &= \frac{d}{dx}(e^{x\ln(a)}) \\ &= e^{x\ln(a)}\frac{d}{dx}(x\ln(a)) \\ &= e^{x\ln(a)}(\ln(a)) \\ &= a^x \ln a \end{aligned}$$

In general, then

$$\frac{d}{dx}a^u = a^u \ln a \frac{du}{dx}$$

Notes

Example

Example (Differentiate $y = 3^{\cos x}$)

$$\begin{aligned}\frac{d}{dx}(3^{\cos x}) &= 3^{\cos x}\ln 3\frac{d}{dx}(\cos x)\\ &= 3^{\cos x}\ln 3(-\sin x)\\ &= -\sin x 3^{\cos x}\ln 3\end{aligned}$$

Example (Differentiate $y = 4^{\tan(x)+x^2}$)

$$\begin{aligned}\frac{d}{dx}(4^{\tan(x)+x^2}) &= 4^{\tan(x)+x^2}\ln 4\frac{d}{dx}(\tan(x)+x^2)\\ &= 4^{\tan(x)+x^2}\ln 4(\sec^2(x)+2x)\end{aligned}$$

Notes

Example

Example (At what point does the tangent line have a slope of 16 if $y = 3^t - 2$?)

Step 1: Find the derivative

$\frac{d}{dt}(3^t - 2) = 3^t \ln 3$

Step 2: Solve $3^t \ln 3 = 16$

Rewrite as $3^t \ln 3 - 16 = 0$ and use your calculator.

Step 3: Calculate the point. If $t = 2.438$ then $y = 3^{2.438} - 2 = 12.562$

Notes

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