## Calculating the Second Derivative

Given $2x^3 - 3y^2 = 8$, find $\frac{d^2y}{dx^2}$

Step 1: Find the first derivative using implicit differentiation.

$$\begin{aligned}
\frac{d}{dx}(2x^3 - 3y^2) &= \frac{d}{dx}(8) \\
6x^2 - 6y\frac{dy}{dx} &= 0 \\
-6y\frac{dy}{dx} &= -6x^2 \\
\frac{dy}{dx} &= \frac{-6x^2}{-6y} = \frac{x^2}{y}
\end{aligned}$$

Notes

## Calculating the Second Derivative (cont.)

Recall

$$\frac{dy}{dx} = \frac{-6x^2}{-6y} = \frac{x}{y}$$

Step 2: Take the derivative of both sides again.

$$\begin{aligned}
\frac{d}{dx}\left(\frac{dy}{dx}\right) &= \frac{d}{dx}\left(\frac{x^2}{y}\right) \\
\frac{d^2y}{dx^2} &= \frac{2xy - x^2\frac{dy}{dx}}{y^2} \\
\frac{d^2y}{dx^2} &= \frac{2x}{y} - \frac{x^2}{y^2} \cdot \frac{dy}{dx} \\
\frac{d^2y}{dx^2} &= \frac{2x}{y} - \frac{x^2}{y^2} \cdot \frac{x^2}{y} \\
\frac{d^2y}{dx^2} &= \frac{2x}{y} - \frac{x^4}{y^3}
\end{aligned}$$

Notes

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## Calculating the Second Derivative (2nd example)

Given $y^2 = x^2 + 2x$, find $\frac{d^2y}{dx^2}$

Step 1: Find the first derivative using implicit differentiation.

$$\begin{aligned} \frac{d}{dx}(y^2) &= \frac{d}{dx}(x^2 + 2x) \\ 2y\frac{dy}{dx} &= 2x + 2 \\ \frac{dy}{dx} &= \frac{2x+2}{2y} = \frac{x+1}{y} \end{aligned}$$

Notes

## Calculating the Second Derivative (cont.)

Recall

$$\frac{dy}{dx} = \frac{x+1}{y} = \frac{x+1}{y}$$

Step 2: Take the derivative of both sides again.

$$\begin{aligned} \frac{d}{dx}(\frac{dy}{dx}) &= \frac{d}{dx}(\frac{x+1}{y}) \\ \frac{d^2y}{dx^2} &= \frac{y - (x+1)\frac{dy}{dx}}{y^2} \\ \frac{d^2y}{dx^2} &= \frac{y}{y^2} - \frac{x+1}{y^2} \cdot \frac{dy}{dx} \\ \frac{d^2y}{dx^2} &= \frac{1}{y} - \frac{x+1}{y^2} \cdot \frac{x+1}{y} \\ \frac{d^2y}{dx^2} &= \frac{1}{y} - \frac{(x+1)^2}{y^3} \end{aligned}$$

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