Finding Slope on a Circle

Example (Find the slope of the tangent line on the circle $x^2 + y^2 = 4$ at $(\sqrt{2}, -\sqrt{2})$)

Step 1: find the derivative

$$\begin{aligned} \frac{d}{dx}(x^2 + y^2) &= \frac{d}{dx}(4) \\ 2x + 2y\frac{dy}{dx} &= 0 \\ 2y\frac{dy}{dx} &= -2x \\ \frac{dy}{dx} &= \frac{-2x}{2y} = \frac{-x}{y} \end{aligned}$$

Step 2: Evaluate the derivative.

$\left.\frac{dy}{dx}\right|_{(\sqrt{2},-\sqrt{2})} = \frac{-\sqrt{2}}{-\sqrt{2}} = 1$

Notes

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Notes

Finding Tangent and Normal of an Ellipse

Example (Find the tangent and normal line of $x^2 - xy + y^2 = 13$ at $(-1, -4)$)

Step 1: Slope of the tangent line - calculate dy/dx

$$\begin{aligned} 2x - (x\frac{dy}{dx} + y) + 2y\frac{dy}{dx} &= 0 \\ -x\frac{dy}{dx} + 2y\frac{dy}{dx} &= -2x + y \\ (-x + 2y)\frac{dy}{dx} &= -2x + y \\ \frac{dy}{dx} &= \frac{-2x + y}{-x + 2y} \\ &= \frac{-2(-1) + (-4)}{-(-1) + 2(-4)} = \frac{-2}{-7} = \frac{2}{7} \end{aligned}$$

Notes

Using the Chain Rule

Example (Find dy/dx for $\sec(xy) = x^2$)

$$\begin{aligned}
\frac{d}{dx}(\sec(xy)) &= \frac{d}{dx}(x^2) \\
\sec(xy)\tan(xy)\frac{d}{dx}(xy) &= 2x \\
\sec(xy)\tan(xy)(x\frac{dy}{dx} + y) &= 2x \\
x\sec(xy)\tan(xy)\frac{dy}{dx} + y\sec(xy)\tan(xy) &= 2x \\
x\sec(xy)\tan(xy)\frac{dy}{dx} &= 2x - y\sec(xy)\tan(xy) \\
\frac{dy}{dx} &= \frac{2x - y\sec(xy)\tan(xy)}{x\sec(xy)\tan(xy)}
\end{aligned}$$

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