Lines: Overview

- What is a line?
- Slope
- Equations of Lines
- Parallel and Perpendicular Lines

What is a Line?

Definition (Mathematical)

A line is a geometric figure formed by a point moving along a fixed direction and the reverse of that fixed direction.

Question

How else can you define a line?

Slope

Definition (Increment)

If a particle moves from point $P_1 = (x_1, y_1)$ to the point $P_2 = (x_2, y_2)$, then the *increments* in its coordinates are $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$

Definition (Slope)

If P_1 and P_2 are two points on a non vertical line, then the slope $m = \frac{\texttt{rise}}{\texttt{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Example

Let $P_1 = (4, -3)$ and $P_2 = (2, 5)$

$\Delta x =$ $\Delta y =$

Slope

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Let $P_1 = (4, -3)$ and $P_2 = (2, 5)$

$\Delta x = 2 - 4 = -2$ $\qquad\qquad$ $\Delta y = 5 - (-3) = 8$

Then the slope of the line going through P_1 and P_2 is

$m =$

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$\Delta x = 2 - 4 = -2$ $\qquad\qquad$ $\Delta y = 5 - (-3) = 8$

Then the slope of the line going through P_1 and P_2 is

$m = \frac{\Delta y}{\Delta x} = \frac{8}{-2} = -4$

Equations of Lines

Definition (Point-Slope Equation)

The equation $\boxed{y = m(x - x_1) + y_1}$ is the point-slope equation of the line through the point (x_1, y_1) with slope m.

Definition (Slope-Intercept Equation)

The equation $\boxed{y = mx + b}$ is the slope-intercept equation of the line through the point $(0, b)$ with slope m.

Example

Write the equation of the line that passes through $(5, -3)$ and has slope $-\frac{4}{5}$:

$y =$

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$y = -\frac{4}{5}(x - 5) + (-3) = -\frac{4x}{5} + (\frac{4}{5})5 - 3 = -\frac{4x}{5} + 1$

Equations of Lines

Horizontal and Vertical Lines

Definition (Horizontal Lines)

A horizontal line going through the point $P_1 = (x_1, y_1)$ has a slope $m = 0$, since $\Delta y = 0$. Its equation is $y = y_1$.

Definition (Vertical Lines)

A vertical line going through the point $P_1 = (x_1, y_1)$ has an undefined slope, since $\Delta x = 0$. Its equation is $x = x_1$.

Example

Write the equation of the horizontal and vertical lines that pass through $(3, -5)$

Horizontal: $y =$

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Write the equation of the horizontal and vertical lines that pass through $(3, -5)$
Horizontal: $y = -5$
Vertical: $x =$

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Example

Write the equation of the horizontal and vertical lines that pass through $(3, -5)$
Horizontal: $y = -5$
Vertical: $x = 3$

Note that horizontal and vertical lines are always perpendicular.

Parallel and Perpendicular Lines

Definition (Parallel)

Two lines are *parallel* if they have the same slope.

Definition (Perpendicular)

Two lines L_1 and L_2, whose slopes are m_1 and m_2 are perpendicular if $m_1 = -\frac{1}{m_2}$.

Example

Given the following lines, find pairs of parallel and perpendicular lines.

1. $y = 3x + 2$
2. $x = 6$
3. $y = \frac{x}{3} - 14$
4. $y = 3x - 4$
5. $y = -3x + 5$
6. $y = 2$

Homework

Section 1.1 exercises (page 9) (12 problems):

2

4

10 a
b

12 a
b

14

20

32 a
b

34 a
b