1 Properties of Logarithms

There is a logical symmetry between the properties of exponential and logarithmic functions, since logarithmic functions are the inverse of exponential functions.

	Exponential	Logarithmic
	$y = b^x$	$x = \log_b y$
Product	$b^m b^n = b^{m+n}$	$\log_b mn = \log_b m + \log_b n$
Quotient	$\frac{b^m}{b^n} = b^{m-n}$	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power	$(b^m)^n = b^{mn}$	$\log_b m^n = n \log_b m$
Change of Base		$\log_b m = \frac{\log_c m}{\log_c b}$

2 Simplifying Logarithms - Examples

Simplifying logarithms usually involves rewriting the expression so that there is one logarithm remaining.

1.

$$\log_3 7 + \log_3 5 = \log_3 35$$

2.

$$\begin{aligned} 2\log_4 x + \frac{1}{2}\log_4 16 &= \log_4 x^2 + \log_4 16^{\frac{1}{2}} \\ &= \log_4 x^2 + \log_4 4 \\ &= \log_4 4x^2 \end{aligned}$$

3.

$$\begin{aligned} \log_7 81 - \log_7 27 &= \log_7 \frac{81}{27} \\ &= \log_7 3 \end{aligned}$$

4.

$$\begin{aligned} 3\log_{14} x - 2\log_{14} x &= \log_{14} x^3 - \log_{14} x^2 \\ &= \log_{14} \frac{x^3}{x^2} \\ &= \log_{14} x \end{aligned}$$

3 Practice Problems

Rewrite each of these expressions as a single logarithm, and then convert to exponential form.

1. $\log_2 x + \log_2 4 =$
2. $\log_2 7 + 3\log_2 4 =$
3. $\log_3 5 + 2\log_3 x =$
4. $\log_b \sqrt{y} + 3\log_b \sqrt{y} =$
5. $\log_{15} y + \log_{15} x + \log_{15} 4 =$
6. $\log_a 7 + 2\log_a 7 + \frac{1}{3}\log_a 7 =$
7. $\log_2 8 - \log_2 4 =$
8. $\log_4 256 - 3\log_4 4 =$
9. $\log_3 5 - 2\log_3 x =$
10. $\log_b \sqrt{y} - \log_b \sqrt{y} =$
11. $\log_{15} y - \log_{15} x + \log_{15} 4 =$
12. $\log_a 7 + 2\log_a 7 - \frac{1}{3}\log_a 7 =$