Calculating the inverse of a quadratic function algebraically

Given a quadratic function $f(x) = ax^2 \pm bx \pm c$ we can use a process called **Completing the Square** to find the inverse.

We use the fact that $(u+v)^2 = u^2 + 2uv + v^2$ and $(u-v)^2 = u^2 - 2uv + v^2$; we let $u = \sqrt{a}x$ and $v = \frac{b}{2\sqrt{a}}$. Then we can rewrite the function $ax^2 + bx + c$ as $u^2 + 2uv + v^2 - v^2 + c$, since $u^2 + 2uv + v^2 - v^2 = ax^2 + bx$.

Examples

$$f(x) = x^2 + 6x - 4$$

1. $y = x^2 + 6x + 4$

2. $$\begin{aligned} x &= y^2 + 6y - 4 \\ &= (y^2 + 6y + 9) - 9 + 4 \\ &= (y+3)^2 - 5 \end{aligned}$$

3. $$\begin{aligned} (y+3)^2 &= x + 5 \\ y + 3 &= \pm\sqrt{x+5} \\ y &= \pm\sqrt{x+5} - 3 \end{aligned}$$

4. $f^{-1}(x) = \pm\sqrt{x+5} - 3$

$$f(x) = 4x^2 - 8x - 5$$

1. $y = 4x^2 - 8x - 5$

2. $$\begin{aligned} x &= 4y^2 - 8y - 5 \\ &= (4y^2 - 8y + 4) - 4 - 5 \\ &= (2y-2)^2 - 9 \end{aligned}$$

3. $$\begin{aligned} (2y-2)^2 &= x + 9 \\ 2y - 2 &= \pm\sqrt{x+9} \\ 2y &= \pm\sqrt{\tfrac{x+9}{2}} \\ y &= \pm\sqrt{\tfrac{x+9}{2}} + 2 \end{aligned}$$

4. $f^{-1}(x) = \pm\sqrt{\frac{x+9}{2}} + 2$

Practice

$f(x) = x^2 - 8x + 12$

1.

2.

3.

4. $f^{-1}(x) =$

$f(x) = 9x^2 + 12x + 7$

1.

2.

3.

4. $f^{-1}(x) =$

To be handed in:

For each of the following functions, calculate the inverse function:

1. $f(x) = x^2 + 6x + 1$
2. $f(x) = x^2 - 12x - 5$
3. $f(x) = x^2 - 4x + 12$
4. $f(x) = x^2 + 10x + 15$

5. $f(x) = 16x^2 + 16x - 5$

6. $f(x) = 9x^2 - 18x + 5$

7. $f(x) = 25x^2 + 30x - 5$

8. $f(x) = 4x^2 - 16x + 5$