This is **NOT** a quiz.

When factoring a polynomial, one approach is to find one of the roots (which we will call r) of the polynomial, then factor out $(x-r)$. The resulting polynomial has a smaller degree, and we can repeat the process again and again until we have the final result.

1 Example: Factoring $x^3 - 6x^2 - x + 30 = 0$

For instance, consider the polynomial $x^3 - 6x^2 - x + 30 = 0$.

- If r is a root, then r must be a factor of 30[1]. Therefore possible values for r are $\pm 1, \pm 2, \pm 3, \pm 5$, $\pm 6, \pm 10, \pm 15, \pm 30$.
- We can test whether a possible root is an actual root by plugging the value in and seeing if the expression becomes 0; 2 is not a root since $2^3 - 6(2)^2 - 2 + 30 = 8 - 24 - 2 + 30 = 12$, but -2 is a root since $(-2)^3 - 6(-2)^2 - (-2) + 30 = -8 - 24 + 2 + 30 = 0$.
- Note that if the root is -2, then the factor is $(x+2)$; specifically, pay attention to the change in sign.
- We calculate $x^3 - 6x^2 - x + 30 \div x + 2$ and get $x^2 - 8x + 15$, with a remainder of zero.
- Now we consider the polynomial $x^2 - 8x + 15$; if r is a root, then r must be a factor of 15. That narrows it down to $\pm 1, \pm 3, \pm 5, \pm 15$.

  As it turns out, 3 is a root, since $3^2 - 8(3) + 15 = 9 - 24 + 15 = 0$.
- Therefore we now calculate $x^2 - 8x + 15 \div x - 3$, and get $x - 5$.
- Since we divided first by $(x+2)$, and then by $x-3)$, and finally got $(x-5)$, we conclude that $x^3 - 6x^2 - x + 30 = (x+2)(x-3)(x-5)$.

2 Sample Problems

Using the same approach, factor the following polynomials. Note that the degree of the polynomial is equal to the number of factors you will get.

(Hint: try the smaller possible roots first.)

1. $x^3 + 6x^2 - x - 30$
2. $x^3 + 4x^2 - 4x - 16$
3. $x^3 + 11x^2 + 16x - 84$
4. $x^4 - 7x^3 + 13x^2 + 3x - 18$
5. $x^3 + 5x^2 - 8x - 12$
6. $x^3 - 3x - 2$
7. $x^3 + 3x^2 + 3x + 1$
8. $x^4 + 3x^3 - 15x^2 - 19x + 30$
9. $x^3 + 3x^2 - 34x + 48$
10. $x^3 + 2x^2 - 39x + 72$
11. $x^4 + 8x^3 + 17x^2 - 2x - 24$
12. $x^4 + 5x^3 - 10x^2 - 80x - 96$

[1]Note that the leading coefficient is 1, and so any root will have 1 as a denominator. If you don't remember or haven't seen the rational root theorem yet, don't worry about it.